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DBJ-003-2015025 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

June – 2022

Physics - 501

(Mathematical Physics, Classical Mechanics & Quantum Mechanics)

(New Course)

Faculty Code : 003
Subject Code : 2015025

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any five questions.

(2) Symbols have their usual meanings.

(3) Figures to the right indicate marks.

Physical Constants :

$h = 6.62\times10^{-34} Js$, $\hbar = 1.055\times10^{-34} Js$, Mass of an electron $= 9.1\times10^{-31} kg$

1 (a) Answer the following objective questions : **4**

(1) In the interval $-l$ to $+l$ the Fourier coefficient $b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}dx$. True or false ?

(2) Write the complex form of Fourier series.

(3) The function $f(x) = x^2$ is an odd function. True or false ?

(4) $\int_{-\pi}^{+\pi} \sin mx \sin nx = \pi$, , when m = ________

(b) Answer the question : **2**

1. Expand $f(x) = x, \ -\pi < x < \pi$.

(c) Answer the question : **3**

1. Explain the action of a half wave rectifier based on Fourier analysis.

(d) Answer the question in detail : **5**

1. What is Fourier series ? Derive Fourier coefficients.

2 (a) Answer the following objective questions : **4**

1. Fourier series can be used for the analysis of pure direct current. True or false ?
2. If $f(x)$ is an even function then $f(x)\sin x$ is ________
3. $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + ...$ True or false ?
4. $\int_{-\pi}^{+\pi} \cos mx \sin nx = ...$

(b) Answer the question : **2**

1. Expand

$$f(x) = 0, -\pi < x < 0$$
$$= 1, 0 < x < \pi$$

(c) Answer the question : **3**

1. Explain the analysis of a triangular wave using Fourier series.

(d) Answer the question in detail : **5**

1. Explain Fourier transform including Fourier cosine transform and Fourier sine transform.

3 (a) Answer the following objective questions : **4**

1. Write D'Alembert's principle.
2. Write the types of time dependent constraint.
3. Write Lagrange's equation of motion.
4. The product of $Q_j \delta q_j$ has the dimension of _______.

(b) Answer the question : **2**

1. Obtain Lagrange's equation of motion for the system having kinetic energy and potential energy, $T = \frac{1}{2} m y^2$ and $V = \frac{1}{2} n \omega^2 y^2$.

(c) Answer the question : **3**

1. Explain D' Alembert's principle.

(d) Answer the question in detail : **5**

1. Deduce Newton's second law from D' Alembert's principle.

4 (a) Answer the following objective questions : **4**

1. Define constraints.
2. What is degree of freedom ?
3. Define Hamilton's variational principle.
4. Write Euler-Lagrangian differential function.

(b) Answer the question : **2**

1. Lagrangian of a compound pendulum is $\frac{1}{2} I \dot{\theta}^2 + mgl\cos\theta$, then what will be its kinetic energy and potential energy ?

(c) Answer the question : **3**

1. Explain virtual work.

(d) Answer the question in detail : **5**

1. Deduce Lagrange's equation from D' Alembert's principle.

5 (a) Answer the following objective questions : **4**

1. Define phase space.
2. Write Hamilton's modified principle.
3. In Hamiltonian formulation n-position coordinates and n-momentum coordinates are considered as independent variables. True or false ?
4. If $\frac{\partial L}{\partial q_j} = 0$, then q_j is considered as ________.

(b) Answer the question : **2**

1. Obtain Hamilton's canonical equations of motion for a system, whose Hamiltonian is given as $H(x, p_x) = \frac{p_x^2}{2m} + \frac{1}{2}kx^2$.

(c) Answer the question : **3**

1. Explain the advantages of Hamiltonian approach.

(d) Answer the question in detail : **5**

1. Obtain Hamilton's canonical equations of motion.

6 (a) Answer the following objective questions : **4**

1. Write Hamilton's canonical equations of motion.
2. If $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}k(x^2 + y^2)$ then $\frac{\partial L}{\partial \dot{x}} = ?$
3. In a conservative system, the potential energy is only ________ dependent.
4. The 2n-dimensional space having n-position coordinates and n-momentum coordinates is known as ________ space.

(b) Answer the question : **2**

1. Lagrangian of a system is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}k(x^2 + y^2).$$

Calculate $\dot{p}_x$ and $\dot{p}_y$.

(c) Answer the question : **3**

1. Explain generalized velocity and generalized momentum.

(d) Answer the question in detail : **5**

1. Obtain Hamilton's canonical equations of motion from Hamilton's variational principle.

7 (a) Answer the following objective questions : **4**

1. The expectation value of momentum is defined as $< P >= \int \psi^* P \Psi d\tau$. True or false ?

2. $\int |\psi|^2 d\tau = N^2, N^2$ is known as ______

3. The minimum frequency of radiation required for photoelectric emission is known as _______.

4. What is photoelectric effect ?

(b) Answer the question : **2**

1. Normalize wave function $\psi(x) = Ae^{ikx}$ over the region $-a < x < a$.

(c) Answer the question : **3**

1. Explain box normalization.

(d) Answer the question in detail : **5**

1. Derive the one dimensional Schrodinger equation and extend it to three dimensions.

8 (a) Answer the following objective questions : **4**

1. The wavelength associated with a moving particle of momentum p is given by λ = ________.
2. Write the three dimensional operator correspondence of momentum.
3. What is Compton effect ?
4. For the conservation of probability $\frac{\partial}{\partial t}\int |\psi|^2 \, d^3x =$ ________.

(b) Answer the question : **2**

1. If $\psi' = ae^{i(kx-\omega t)}; -1 < x < 1$, obtain the normalized wave function.

(c) Answer the question : **3**

1. Obtain the time independent Schrodinger equation.

(d) Answer the question in detail : **5**

1. Describe a square well potential and explain bound state $(E < 0)$.

9 (a) Answer the following objective questions : **4**

1. $[x, P_x] =$ _______.
2. What is a self adjoint operator ?
3. If A and B are adjoint operators then $(A+B)^{\dagger} = A^{\dagger} - B^{\dagger}$. True or false ?
4. In quantum mechanics each dynamical variable is represented by a linear operator. True or false ?

(b) Answer the question : **2**

1. Prove that momentum operator is self adjoint.

(c) Answer the question : **3**

1. Write a note on Dirac Delta function.

(d) Answer the question in detail : **5**

1. State and explain the fundamental postulates of wave mechanics.

10 (a) Answer the following objective questions : **4**

1. $[L_x, L_y] =$ ______.
2. The ground state energy of a harmonic oscillator = ______.
3. $(A^{\dagger})^{\dagger} =$ ______.
4. If $\delta(x - x')$ is the Dirac delta function then, $\int_{-\infty}^{+\infty} \delta(x - x') dx =$ ______.

(b) Answer the question : **2**

1. Show that the expectation value of a self adjoint operator is real.

(c) Answer the question : **3**

1. Obtain the Schrodinger equation for a simple harmonic oscillator.

(d) Answer the question in detail : **5**

1. Explain angular momentum operator. Derive the expressions for L_x, L_y and L_z and hence obtain the relation $L^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right]$.